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AUCTIONING OFF THE AGENDA: BARGAINING IN LEGISLATURES WITH ENDOGENOUS SCHEDULING

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SOCIAL SCIENCE WORKING PAPER 1266

March, 2007

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Abstract

There are many examples of allocation problems where the final allocation affects more than one agent, but the models developed to study them typically allow for side payments between agent. However, there are political economy applications where it is hard to imagine monetary transfers between the agents, at least not legal ones. In this paper we propose a general political economic framework for the study of allocation problems with externalities without side payments. We consider a setup with complete information and we formulate the problem as one where the status quo describes an initial allocation that can altered in a sequence of *proposals*. The number of these proposals is restricted. In the context of our main application, bidding for slots on a legislative agenda, such restriction can be interpreted as scarcity of plenary time for considering the possible bills to move the policy. The intuition for our model comes out of framing the problem as a special type of a multi-good auction. We show that equilibria generically exist within the general model.

1. INTRODUCTION

There are many examples of allocation problems where the final allocation affects more than one agent. For example, auctions where the allocation of the good imposes an externality on the buyers who do not obtain the good or in all-pay contests.[1] The

*We thank Douglas Bernheim, Gary Cox, Matias Iaryczower, Matthew Jackson, Preston McAfee, Barry Weingast, and William Zame as well as seminar participants at the Hoover Insitution, the University of California, Berkeley and the University of California, San Diego for helpful comments. Čopič acknowledges the financial support of the Social and Information Sciences Laboratory at Caltech and the Cowles Foundation for Research in Economics at Yale University and Katz acknowledges the financial support of the Center for Advanced Study in the Behavioral Sciences.

[1] Auctions with externalities have been studied by McAfee and McMillan (1992), Jehiel et al. (1996), Jehiel et al. (1999). For recent work on all-pay contests with complete information see Siegel (2006).

literature on allocation problems with externalities has focused mainly on the question of mechanism design such that the seller can maximally extract the available surplus. An important feature of all of the models in this literature is that there can be side payments between the agents to compensate for the imposed externality. Indeed, optimal mechanisms often involve transfers between a large subset of the agents. However, there are political economy applications where it is hard to imagine monetary transfers between the agents, at least not legal ones. Consider, for example, a legislature moving policy from the status quo to some other point in the policy space. Even in more traditional economic examples, one can easily conceive of environments in which the agents are risk averse with respect to monetary payments, and such examples cannot be analyzed with the tools of the existing literature.

In this paper we propose a general political economic framework for the study of allocation problems with externalities without side payments. We consider a setup with complete information and we formulate the problem as one where the status quo describes an initial allocation that can be altered in a sequence of *proposals*. The number of these proposals is restricted. In the context of our main application, bidding for slots on a legislative agenda, such restriction can be interpreted as scarcity of plenary time for considering the possible bills to move the policy. In the context of allocation of goods, which we do not consider, the interpretation of such a restriction would be simply that there is a finite number of goods to be allocated.

This feature that plenary time — the time consumed in plenary session when important bills are considered by formally stated motions that must be voted upon – is scarce, is an ubiquitous feature of all legislatures (Cox 2006).[2] In fact, this scarcity has been argued to lead to many of the structures we see in modern legislatures (Cox 2006, see also Polsby, Gallaher, Rundquist 1969). Yet, no models of legislative bargaining explicitly include this scarcity. The standard models assume what we will call *independent scheduling*. That is, the likelihood that a bill is considered by the full chamber is independent of its content and the other bills proposed. For example, in Shepsle (1979) every committee is a monopoly supplier of proposals in their given jurisdiction with guaranteed access to a floor vote for their bill. Baron and Ferejohn (1989) allow that some agents may not necessarily be able to make proposals, but the likelihood that a particular agent can make one does not depend on her actual proposal. The Baron and Ferejohn (1989) model has been extended to allow for endogenous proposal probabilities, but these probabilities are determined *ex ante*, given a member's seniority (McKelvey and Reisman 1992). Yet, no modern legislature works this way. For example, in the U.S. House of Representatives scheduling matters are handled by the Rules Committee, arguably the most powerful committee in the chamber. The Rules Committee is well aware of a bill's content, as well as any proposed amendments, before it allows it to be scheduled for a floor vote.[3]

[2]For example, it is not possible that a legislature could meet in a plenary session for more than 24 hours a day and often there are constitutional limits on the number of days a given legislature can be in session.

[3]In fact, the Rules Committee also gets to choose the rules for the vote, which gives it even more power.

We thus develop a general model of legislative scheduling with scarcity, framing the problem as a special type of a multi-good auction. Here the "goods" to be auctioned off are the limited slots for floor votes and some agent is charged with choosing which bills will be chosen for votes. This scarcity induces competition for these limited slots. However, the situation is more complicated than in a standard auction, because agents can only bid in terms of policy proposals and they all care about the finally enacted policies. At the same time, no side payments are allowed. In a standard auction, agents care about the bids of other players only insofar as it affects the price they must pay for the good, and in the aforementioned literature on auctions with externalities, side payments are allowed between the agents. In the legislative situation, on the other hand, if another agent's proposal is chosen, it will change the set of final enacted policies and, therefore, all agents' payoffs. Thus, there are built-in externalities within the model. Given that in most democracies contracting on side payments with politicians is illegal, it will be difficult to internalize and price these externalities. Even with this difficulty, we show that equilibria generically exist within the general model.

While our general model provides a very flexible framework that can be extended in many directions, we add some details in order to make interesting comparative-statics statements. We do so in what we think of as the simplest case of complete-information legislative bargaining. We use our framework to generalize Shepsle's (1979) model. In our model of legislative bargaining, competition for access to the floor induces a final policy to move towards the policy preferences of the scheduling agent, even if the policy preferences of the proposers are aligned and contrary to the scheduling agent's preferences.

In Shepsle's (1979) view of committees, which has dominated the literature, as monopoly suppliers of policy proposals, there are serious agency problem within the legislature because the committees can use their agenda power to extract all the surplus from the majority (see also Denzau and Mackay 1983).[4] The conventional wisdom was that the only real way to ameliorate this agency loss was by screening. This lead to a voluminous empirical literature on whether or not legislative committees are preference outliers (see, for example, Cox and McCubbins 1993, Londregan and Snyder 1994, Poole and Rosenthal 1997). Our model, however, suggest a new way to think about this agency problem. When we include competition for plenary time in the model, this competition can prevent the agency loss if the scheduling agent is chosen correctly. Nevertheless, this may create another agency problem in the choice of the scheduler.

As noted above, the legislative scheduling problem has not really been studied. The most directly related work is by Cox and McCubbins (1993), who model the scheduling process under incomplete information as a multi-armed bandit problem. However, their model is non-strategic and they do not examine the impact of competition on proposal behavior. They instead examine the order the scheduling agent would have proposals voted on. McKelvey and Riezman (1992) take a totally different approach whereby the recognition probability of a member is determined by a seniority rule in order to

[4]Gilligan and Krehbiel (1990) also generate agency problems with legislative committees but because of informational asymmetries; see also Krehbiel (1992).

endogenously generate an incumbency advantage for the members of the legislature, but they assume independent scheduling. There have also been a number of papers on endogenous agenda formation, such as Banks and Gasmi (1987) and Penn (2005), but these are best characterized as determining the amendment tree of a single bill and again assume independent scheduling of the final votes.

The rest of this paper is organized as follows. In Section 2 we provide a very simplified example of our model to give some guiding intuitions. In Section 3 we give the general framework for our political economy model, the *scheduling game*. In Section 4 we apply this framework to a more specific model of legislative voting game, and we provide a comparative-static result for that game. In Section 5 we give some additional insights into the workings of the legislative model. In Section 6 we examine the impact of scheduling on assignment of committee jurisdictions. Finally, in Section 7 we discuss other potential applications of our model, as well as possible extensions to incomplete information.

2. MOTIVATING EXAMPLE

In this section we illustrate the basic intuitions of our approach with examples of outcomes of a very simple case of the *legislative game*. We show how competition for slots on the agenda affects the proposed bills and the outcomes of the legislative process. We consider a simple setup with 2 committees (voters) and *Speaker*, the agenda-setting agent.[5] In the first stage of the game the agenda is determined (either by Speaker, or randomly), and in the second stage the *status quo* is voted against the whole agenda in one up or down vote. We compare the outcomes under legislative competition to two benchmarks: no competition, where there are enough slots to accommodate all proposals; and independent scheduling of slots under a budget constraint on the number of slots, where the likelihood of getting a proposal on the agenda is independent of the content of the proposal. These comparisons highlight the effect of Speaker's agenda-setting power.

There are 3 agents, $\mathbf{N} = \{0, 1, 2\}$. Agent 0 is a principal, *Speaker*, and agents 1 and 2 are the committees, who on the one hand make proposals, and on the other hand they vote between the final proposed move and the *status quo*. The outcomes are represented by $\Re^2$, and $\mathbf{0} = (0, 0) \in \Re^2$ is the *status quo*. Each voter $i \in N$ has quadratic preferences over outcomes, with an ideal point $y_i \in \Re^2$. The utility function of agent i, $u_i : \Re^2 \to \Re$, is thus given by

$$u_i(\mathbf{x}) = -||\mathbf{x} - y_i||^2 = -(x_1 - y_{i,1})^2 - (x_2 - y_{i,2})^2, x \in \Re^2,$$

where $||.||$ denotes the Euclidian norm on $\Re^2$. The *length of the agenda* is denoted by T, $T \in \{1, 2\}$. For the rest of this section we assume that $y_0 = (-1, -1)$, $y_1 = (1, k)$,

[5]We choose the term *Speaker* because this is the title of person in control of the schedule in the U.S. House of Representatives. However, all democratic legislatures that we are aware of delegate scheduling to some member or committee — such as a prime minister, cabinet, chairperson, or president.

$1 \geq k \geq 0$, and $y_2 = (1, 2)$.The examples here do not hinge on precise parameter values, but this parametrization allows us to illustrate the analysis.

The *legislative game* is as follows.

- In stage 1, each voter $i = 1, 2$ proposes a move from the *status quo* on dimension i.
- In stage 2, *Speaker* must select exactly T of those proposals, e.g., if $T = 1$ then *Speaker* has to make the choice between one or the other proposal.
- In stage 3, voters and Speaker vote (by majority) between implementing either the *status quo* or the move from the *status quo* given by the combined selected proposals.

We study the subgame-perfect Nash equilibria (SPNE) of this game.

The first stage corresponds to a process where committees have jurisdictions and are only allowed to propose moves from *sq* within their jurisdiction. In the second stage, the assumption that Speaker has to select precisely T proposals corresponds to having *no gate keeping power* in the agenda-formation process. Finally, in Stage 3, *Speaker* only has a tie-breaking vote - in an example with an odd number of voters he would never vote, and with an even number of voters he votes when there is a tie.

We start with the benchmarks of no competition and independent scheduling. In both of these cases, the incentives of the proposers 1 and 2 are very similar: a proposer just makes a proposal that will make the agenda optimal for him conditional on being acceptable to the majority (or unacceptable to every majority, if the proposer prefers *sq* to every other majority-feasible outcome).

No competition, T $= 2$. Since Speaker must choose both proposals the only constraint that the proposers have to consider is the majority-voting of Stage 3 - effectively, the game only has two stages (1 and 3). There are two SPNE outcomes. One is the *status quo*. This obtains for example if each proposer proposes a very extreme move which makes by itself *status quo* the preferable outcome to all agents (this requires the use of weakly dominated strategies). The unique SPNE outcome other than the *status quo* is the point $\left(1, \min\{\sqrt{1+(1+k)^2}, 2\}\right)$, $\forall k \in [0, 1]$, where in every voting subgame voter 1 votes for the move whenever indifferent between the move and the *status quo*. We show this for $k = 0$, the logic for $k \in (0, 1]$ is exactly the same. First, it is enough to consider proposals m_i, such that $m_i \leq y_{i,i}$, $i = 1, 2$ - a voter never has an incentive to propose a move that is further than the ith component of his ideal point. Since $k = 0$, proposals $m_1 = m_2 = 1$ constitute mutual best replies, when in the voting stage voter 1 always votes for the move whenever indifferent between *status quo* and the combined move (m_1, m_2). Next, for every proposal m_1 of voter 1, voter 2's best reply is to propose an m_2 such that (m_1, m_2) is on 1's indifference curve through $\mathbf{0}$. But 1's best reply is then to either propose something on 2's indifference curve through $\mathbf{0}$ or to propose a move of size 1, whichever of the two is smaller. Indeed, $(1,1)$ is the unique SPNE outcome different from the *status quo*. Note that if ideal points are $y_1 = (1,0), y_2 = (0,1)$ the set of Pareto-optimal outcomes is $(s,s), s \in -1, \frac{1}{2}]$, and only the *status quo* so that the outcome $(1,1)$ is not Pareto-optimal.

Independent scheduling, $\mathbf{T} = 1$. The bid of each voter now reflects only that conditional on his proposal being selected, the proposal has to just beat *status quo* in the voting stage. The equilibrium proposals of the two proposers are

$$m_1 = 1, m_2 = 2k.$$

Agent 1 bids his ideal point on dimension one, since that is preferred to the *status quo* by a majority consisting of 1 and 2. Agent 2 bids the point that makes 1 just indifferent between 2's proposal and the *status quo*. The equilibrium outcomes are then either $(1,0)$ or $(0,2k)$, and which of the two obtains depends on the specifics of the independent-scheduling protocol – e.g., seniority ordering, or any ordering given by some other exogenous variable or process.

Agenda competition, $\mathbf{T} = 1$. Due to scarcity of slots, each proposer may have an incentive to outbid the other in the bidding stage. There are 2 cases. If $k < 1$ then the unique equilibrium outcome is $\mathbf{0}$. In Section 5 we provide simple conditions which assure that *status quo* is the unique equilibrium outcome. If $k = 1$ then these conditions are not satisfied, and indeed each outcome $(0,s)$, $s \leq 1$, can be supported as an equilibrium outcome. In the following paragraphs we show these claims directly.

<u>Case 1: $k < 1$.</u> For every proposal $m_j > 0$ of proposer j, there exist proposals $m_i < m_j$ for proposer i, such that the move of the size m_i in the direction i makes i better off than a move of the size m_j in the direction j. Figure 1 illustrates this. It depicts the situation where $k = \frac{1}{2}$, so that $y_1 = (1, \frac{1}{2})$, and the lines l_1 and l_2 show the distance from y_1 to the outcomes that are obtained by a move of 0.5 on either of the two dimensions. *Speaker* is indifferent between these two moves, while proposer 1 prefers the move on dimension 1. Lines l_3 and l_4 indicate the distances to 2's ideal point from the outcomes under the two moves: 2 prefers the move on dimension 2. Since m_i then gets selected by *Speaker* (because $m_j > m_i$), it follows that at least one proposer proposes $m_i = 0$, and the unique equilibrium outcome is $\mathbf{0}$.[6] It is easy to see that the crucial geometric condition here is that the ideal points of the proposers lie on different sides of the line $x_1 = x_2$. This condition is a simple version of sufficient conditions assuring *status quo*, which we provide Section 5.

<u>Case 2: $k = 1$.</u> Between two equidistant moves (less than 1) along either of the axes both voters now weakly prefer (and voter 2 strictly prefers) the move along axis 2, which

[6]For instance, if 1 proposes $m_1 = s, 0 < s < 1$, then the utility of 2 under such move is $u_2(s,0) = -(1-s)^2 - 2^2$, while $u_2(0,s) = -1^2 - (2-s)^2 < u_2(s,0)$, so that there indeed exist proposals m_2 by 2, such that $m_2 < s$ and $u_2(0,m_2) > u_2(s,0)$. Note that if $k = 0$ then a positive move along the axis 2 cannot get a majority support anyway.

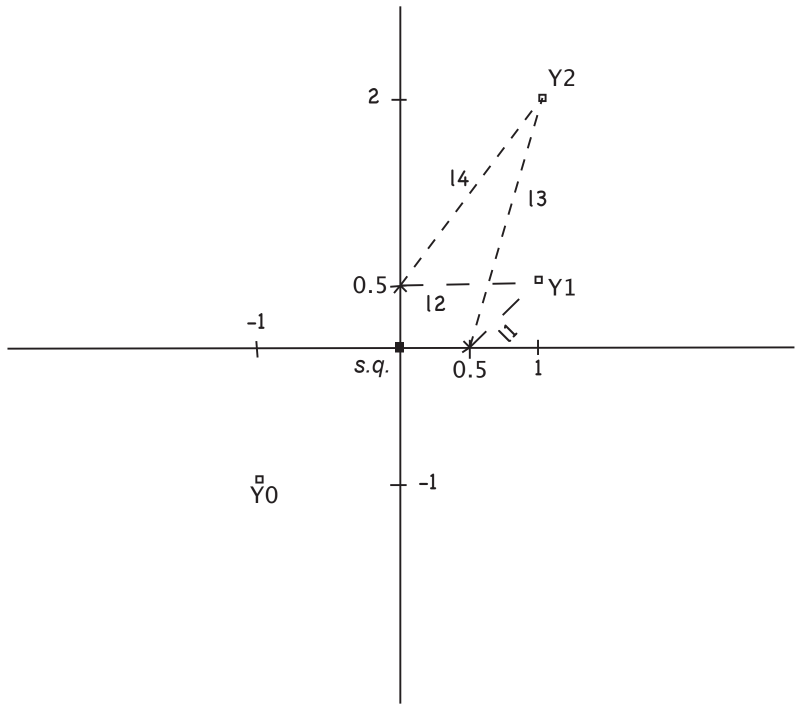

Figure 1: An example of the three-person legislative game with agenda competition and $k = \frac{1}{2}$. Given that $k < 1$, the unique equilibrium outcome is $\mathbf{0}$ as each voter has an incentive to "outbid" the other.

facilitates the set of equilibrium outcomes. Figure 2 illustrates 1's indifference between the moves for 0.5 on either of the two axes. The lines denoted by l_1 and l_2 (which are now of equal length) show the distance from his ideal point y_1 to the attained outcome under each of the two moves. Thus, if both agents propose $m_i = s$ and *Speaker* selects m_2, this is an equilibrium with the outcome $(0, s)$ – *Speaker* has to resolve his indifference in precisely the right way - by picking 2's proposal when indifferent. Similar result obtains for $k > 1$.

We stress a few observations to which we return in subsequent sections. First, the equilibrium always exists. Under *no competition* or *independent scheduling* this follows from standard arguments. However, in the *legislative game*, under competition standard arguments are invalid as it is a game of discontinuous payoffs, and best-reply correspondences are empty at many proposals. These problems can be sidestepped by determining *Speaker's indifference-breaking rule as a part of equilibrium*, it may be different at different points, and it can then be shown that the equilibrium then exists in the general

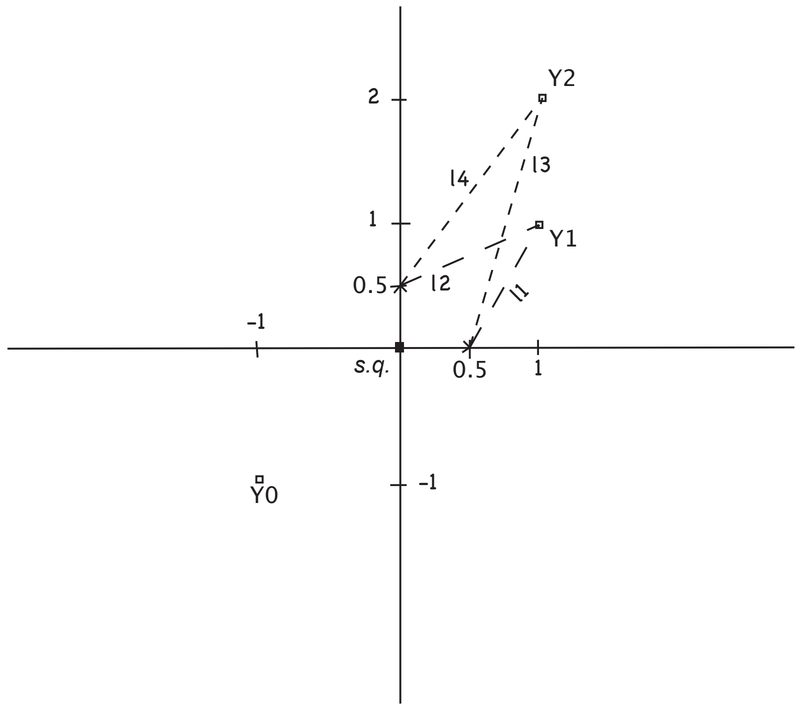

Figure 2: An example of the three person legislative game with agenda competition and $k = 1$. In this case any outcome $(0, s)$, $s \leq 1$, can be supported as an equilibrium. The picture shows the case with $s = 0.5$.

model.[7] A feature of the legislative game is that Speaker's indifference-breaking rule is generically deterministic – it might still be that at different points in the space of bids, Speaker resolves his indifference in favor of different proposers.

The second observation concerns the comparison of equilibrium outcomes under competition with those under *independent scheduling*. The set of outcomes under competition moves closer to *Speaker*'s ideal point. This is apparent in the above example, and the intuition is that under competition agents have incentives to make proposals that are closer to *Speaker*'s ideal point; under *independent scheduling* no such incentives exist. In Section 4 we prove a general comparative-static result of this sort.

While one may think that the comparison with *no competition* works in a similar way, this turns out not to be the case. Then, there are two effects which work in opposite directions: if the preferences of all the agents are closely aligned with Speaker's then having more slots might give more possibilities to move the outcome closer to Speaker's

[7] This idea comes from Simon and Zame (1990) and we use their main Theorem 1 to prove existence here.

ideal point, while if agents' preferences are less aligned with Speaker's then having more slots will decrease competition and move outcomes further away from Speaker's ideal point. This comparison is therefore in general specific to the preference parameters of the agents, and we formalize this with a simple example in Section 4. In Section 5 we show that even under fixed preferences which are not aligned with Speaker's it is not necessarily the case that fewer slots move the outcome closer to Speaker's ideal point.

3. THE SCHEDULING GAME

In this section we present the general model which we call the *scheduling game*. This provides a much more general framework than the introductory example of Section 2. In Theorem 1 we show the existence of an equilibrium of the scheduling game.

The set of agents is $N = \{0, ..., n\}$. Agents $1, ..., n$ are proposers (or voters), and agent 0 is the *agenda setter* (or *Speaker*). The set of possible outcomes is M, where M is some metric space. There is a *status quo*, $m_{sq} \in M$, without loss of generality assume that $m_{sq} = 0$. Agents' preferences over points in M are rational and are given by $\succeq = (\succeq_0, ..., \succeq_n)$. We assume that for each i, $\succeq_i$ is representable by a continuous function $u_i : M \to R$.

Scheduling game proceeds as follows. In the first stage each agent $i \in N \setminus \{0\}$ makes a proposal, $m_i \in M_i$, where M_i is a compact subset of M. We call M_i *the jurisdiction* of proposer i. Proposers submit their proposals simultaneously. The number of proposals that can be considered is limited by $T \leq n$. We call T *the length of the agenda*, and it is exogenously specified. In the second stage, agent 0 picks T proposals, $L \subset N \setminus \{0\}, |L| = T$.[8] In the third stage, the final outcome is selected by voting between the *status quo* and the outcome given by the selected proposals. Formally, denote by $V_q(x, y)$ the q-majority voting correspondence, i.e.,

$$V_q(x,y) = \begin{cases} x & \text{if } \exists N' \subset N \text{ s. t. } \frac{|N'|}{|N|} \geq q \text{ and } x \succeq_i y, \forall i \in L, \\ y & \text{if } \exists N' \subset N \text{ s. t. } \frac{|N'|}{|N|} \geq q \text{ and } y \succeq_i x, \forall i \in L, \end{cases}$$

where $q \geq \frac{1}{2}$, and $x, y \in M$. The final outcome of the third stage is then given by

$$a_q(m, L) = V_q(0, \sum_{t \in L} m_t).$$

We remark that our model of the legislature is in reduced form. That is, in practice, we want to think of one period as an entire legislative session. For example, the session

[8] As we noted in Section 2, one could also consider an extension of this model where 0 can pick less then T proposals, but in the present work we limit ourselves to the case when 0 *must* choose T proposals. Even in the present scenario the final outcomes will be very favorable to the agenda setter relative to the case without agenda competition.

of the U.S. Congress is 2 years. During this period, the outcome is then moved from the status quo vote by vote, rather than in one *combined move*, as in our model. First, in a complete-information environment, the equilibria in our model will subsume the equilibria of a sequential model. Second, showing equilibrium existence can easily be extended to a sequential model. Nevertheless, a sequential model will be much more difficult to analyze, and our results provide a useful starting point.

We interpret the present model as one where agents $1, ..., n$ are committees, and M_i are their jurisdictions, so that the committee i can only propose a move from the *status quo* in her jurisdiction M_i. In this simplest setup, each committee is one voter: we abstract from the fact that committees may be composed of many voters and the proposed moves may themselves be a result of some voting or negotiation process. Agent 0, *Speaker*, has monopolistic power over the agenda, and picks among the proposals that he likes most.[9]

We consider the subgame-perfect Nash equilibria (SPNE) of the *scheduling game*. To be precise, a SPNE is given by a vector of proposals, $m^* \in \times_{i \in N} M_i$, and a mapping $L^* : \times_{i \in N} M_i \to \{L \mid L \subset N, |L| = T\}$, such that

$$a_q(m^*, L(m^*)) \succeq_0 a_q(m^*, L), \forall L \subset N, |L| = T,$$
$$a_q(m^*, L(m^*)) \succeq_i a_q(m^*_{-i}, m_i, L(m^*_{-i}, m_i)), \forall m_i \in \times_{i \in N} M_i, \forall i \in N.$$

Thus, m^*_i is the proposal of proposer i, and, for each $m \in \times_{i \in N} M_i$, $L^*(m)$, is the set of proposers selected by the speaker. An outcome of a SPNE is determined by m^* and $L^*(m^*)$; we will often abuse the notation and denote $L^*(m^*) = L^*$.

In the *scheduling game*, existence of SPNE is not obvious. The difficulty is that best-reply correspondences of agents $N \setminus \{0\}$ can be empty at many points - the game has similar features as a Bertrand game in the sense that an "under-cutting" of proposals occurs at proposals m that are out of equilibrium. In particular, the set of proposals that a voter may deviate to is generically open, hence no best-reply exists at many points out of equilibrium.

To illustrate this recall the *agenda competition* of Section 2, and consider the situation when $k = \frac{1}{2}$, so that the ideal points of the proposers are $y_1 = (1, \frac{1}{2})$ and $y_2 = (1, 2)$. Then, if for instance proposer 1 proposes $m_1 = \frac{1}{2}$, the best reply to such proposal by agent 2 is $m_2 = \max(0, \frac{1}{2})$ which does not exist.

The way around these problems is by use of Simon and Zame (1990) result. In our case their result implies that if the indifference-breaking rule of agent 0, when faced with a vector of proposals $m = (m_1, ..., m_n)$, is determined in equilibrium, then such equilibrium always exists. More precisely, we use backward induction and show that the subgame-perfect equilibrium correspondence from the first-stage proposals of agents $N \setminus \{0\}$ into outcomes, $Q : M_1 \times M_2 \times ... \times M_n \to M$, satisfies the assumptions of Simon and Zame

[9] Our analysis can also be extended to the case where agent 0 is a dictatorial auctioneer, and there is no voting stage, i.e., agent 0 just picks T proposals and this determines the final outcome. Such model is applicable to examples of auctions with interdependent preferences.

(1990) if the indifference-breaking rules in subsequent stages are left to be specified in equilibrium. Thus, a SPNE of the game exists. The proof of the following theorem is in the Appendix.

Theorem 1. The set of SPNE of the Scheduling game is non-empty.

4. THE *LEGISLATIVE GAME* AND COMPETITION

The purpose of this section is to demonstrate the richness of our framework in a relatively simple setup of the *legislative game* which we already introduced in Section 2. Our main general result is a comparative-static comparing across two different institutions, one of them being the legislative game. Another possible comparative-static exercise is the comparison along different constraints in terms of the number of issues that can be considered.[10] In this section we demonstrate with a very simple example that this second comparative-statics exercise depends heavily on the specific parameters of the model even in the simplest comparison when the time constraint either is binding or it is not binding at all. In the next section, we return to this second exercise. There we show in a specific setup, that the outcomes can move non-monotonically with the number of proposals that can be considered.

The legislative game is defined as the scheduling game where $M = \Re^{|N|}$ and for each $i \in N$,

$$M_i = \{m_i \mid m_i \in \Re^{|N|} \cap [-K, K]^{|N|}, m_{i,j} = 0, \forall j \neq i\},$$

where $K >$ is some large constant. The preferences of each agent $i \in N$ are quadratic with an ideal point $y_i \in \Re^{|N|}$, that is,

$$u_i(x) = -\sum_{j=1}^{N} (x_j - y_{i,j})^2, i \in N.$$

As in the general scheduling game, the number of slots on the agenda is $T \leq |N|$.

Remark 1. It follows directly from Theorem 1 that an equilibrium of the legislative game always exists. What is particular to the legislative game is that the indifference-breaking rule of Speaker is in equilibrium generically deterministic, at each point of the space of proposals. This follows from strict quasi concavity of preferences so that if Speaker randomized between selecting proposals of two different proposers, at least one of the proposers would in a generic situation be able to bid slightly closer to Speaker's ideal point and be selected for sure. By generic we mean that the set of collections of proposers' ideal points for which this logic does not hold has a Lebesgue measure zero.

[10] In the present model, the number of slots can be thought of as a representation of the physical constraints faced by the legislature. Such constraints might be either simply technological (e.g., the amount of paperwork that can be processed), or institutional, as the rules of the institution might impose more or less friction on the legislature and thus affect the number of proposals that can be considered.

To introduce the legislative game, we provide a simple example of a well-known legislative strategy of killing a bill by appending an unappealing amendment. Aside from this positive observation, this example demonstrates very clearly the issues that arise in the equilibrium analysis of the legislative game. In the configuration of this section, if the outcome is non-zero, it must be that all agents $i = 1, ..., n$ prefer that outcome to *sq*. For if the Speaker has only one "accomplice" (who prefers status quo to given proposals), that will be enough to force the zero outcome: that proposer can *kill the bill* by proposing far-fetched proposals which the speaker puts in his selection in order that *sq* be preferred by the majority.

Example 1 (Kill Bill)**.** Let $N = \{0, 1, 2, 3\}$, i.e., $n = 3$, $y_0 = (-1, -1, -1)$, $y_1 = y_2 = (1, 1, 0)$, $y_3 = (-1, -1, 0)$, and $T = 2$ (note that there are 3 voters so that *Speaker* doesn't vote but only selects the agenda since there is never a tie-breaking situation). Then 0 is the unique equilibrium outcome, and it is not supported by agent 3 proposing 0, but for example by proposals $m_1 = m_2 = 1$ and $m_3 = 10$.

We now consider the first comparative static across institutions. The comparison here is with the institution that we call *Independent Scheduling* (IS),

Definition 1 (Independent Scheduling)**.** First, a deterministic ordering L^I of proposers is exogenously specified. Next, the proposals of the first T proposers are selected. Finally, the outcome is determined by voting between the *status quo* and the vector of selected proposals.

Under IS, *Speaker* plays no role (except for having the tie-breaking vote). Denote by $m^{*I}(L^I)$ the SPNE vector of proposals of the voters under IS, given the exogenous ordering is L^I.

In the next proposition we provide a comparative-statics result showing that competition is an important force driving the outcomes. This proposition shows that *Speaker* is better off in the legislative game than under IS. That, is, given an equilibrium outcome under IS, there exists an equilibrium outcome of the legislative game, where the speaker is weakly better off.

Briefly, the idea of the proof is as follows. Take an equilibrium x^* under IS. Then define a new agenda game, where all voters have strategy sets such that they can only propose at least as good things for the speaker as under x^*. By the existence theorem there exists an equilibrium x^{**} of that auxiliary agenda game. Finally, one can show that x^{**} is also an equilibrium of the original agenda game, and it is clearly at least as good for the speaker as x^*. This last step takes a bit of care, and we refer the reader to the Appendix for the details.

Proposition 1. Let y_i, $i \in \mathbf{N}$, be the ideal points of the agents, and let $T \leq n$. Then, for every equilibrium outcome m^{*I} under IS there exists an equilibrium outcome of the legislative game, m^{*A}, such that $m^{*A} \succeq_0 m^{*I}$.

The second benchmark is the case of no competition, i.e., the legislative game with $T = |N|$. In this case, independent scheduling and the legislative game coincide because the speaker selects all proposals anyway.[11] One might be tempted to believe that in general, under no competition *Speaker* is worse off than when $T < |N|$ in the legislative game. However, as the following simple example demonstrates, this is not necessarily true even in this simplest case when we only compare between the case when there is a constraint on the number of proposals that can be considered and the case when there is no such constraint. Here we show that such comparison fundamentally depends on the specification of parameters. In the next section, we show that even when parameters are *fixed* and we want to compare between different constraints, such comparisons depend on precise scheduling constraints.

Example 2. Let $y_0 = (-1, -1, ..., -1), y_i \geq 0, i = 1, ..., n$. Denote $m_y = (y_{1,1}, y_{2,2}, ..., y_{n,n})$, and assume that for each $i = 1, ..., n$, y_i is such that $a_q(m_y, N) = m_y$, so that in particular, $m_y \succeq_i 0$, $i = 1, ..., n$. Now let $T < |N|$ and let $T' = |N|$. Under the present assumptions, the unique equilibrium outcome under no competition (under T') is m_y. On the other hand, 0 is an equilibrium outcome under T, which is very easy to check. Clearly, that outcome under T is better for Speaker. Now suppose that all the agents have the same ideal point $y_0 = y_i, \forall i = 1, ..., n$. Then under T' unique equilibrium outcome is y_0, while under T, equilibrium outcome is different from y_0, as long as $y_{0,j} \neq 0, \forall j$. In words, if all the agents agree about what the outcome should be, then they are all better off if there is enough time to move to that outcome.

5. COMPETITION AND DEADLOCKS

In this section we examine more closely the outcomes of the *legislative game* relative to the number of slots that are available, for a relatively specific set of agents' preferences, as parametrized by their ideal point. We look at a situation, in which *Speaker* has extreme preference relative to the rest of the legislature: i.e., she would like to move the outcome from the *status quo* into the opposite direction than the rest of the legislature. More precisely, we assume that $y_0 = (-1, -1, ..., -1)$, and $y_i \geq 0, i = 1, ..., n$. In such a situation, a *deadlock* on the *status quo* may obtain as a result of the competition between committees for slots.

We do not necessarily believe this situation to be realistic. In fact, if the scheduling agent, as well as the rules of the legislature, are first chosen (or can be subsequently changed) by a simple majority vote, this configuration does not seem a very likely one. A majority of the legislature would be better off with either a different choice of *Speaker* or a different scheduling regime.

However, focusing on the present case of preference specification is reasonable since every deadlock situation that can be assured by *Speaker* alone can also be assured when not all the committees are outliers relative to *Speaker*'s preference (recall Example 1). Under a different preference configuration, where for instance *Speaker*'s preference is

[11] In this case, as well as under independent scheduling, the existence of equilibrium in the corresponding game follows easily by standard arguments.

more aligned with the majority, deadlock situations will not be that common. The outcome will then just move closer to *Speaker*'s ideal point. The present environment is particularly simple for illustrating how *Speaker*'s power matters.

It is somewhat surprising to us that under more competition, smaller T, it is not necessarily easier to obtain a deadlock situation. This provides a further insight into how the set of equilibrium outcomes might change as we vary the constraint on the number of slots on the agenda.

From Section 2 recall that what matters for obtaining a deadlock when $n = 2$ is that proposers' ideal points lie on different sides of the diagonal. A general version of this condition plays a role with $n > 2$ proposers, and is formulated as follows.

Definition 2 (Competitive Condition)**.** A pair of proposers $i, j \in N, i \neq j$ satisfies Competitive Condition (CC) if $y_{il} < y_{ik}$ and $y_{jk} < y_{jl}$, where $\{i, j\} = \{k, l\}$.

The intuition behind CC is that two agents with ideal points on the opposite sides of the diagonal (in the subspace of their proposals) will undercut each-other's proposals all the way to *status quo*, so that the resulting outcome of the legislative game is a deadlock. In the present setup (see Example 2 from Section 4), some competition is better from *Speaker*'s perspective than no competition. Here we show that even if the proposers' ideal points are almost perfectly aligned, but CC is satisfied for sufficiently many pairs of agents, then *status quo* is the unique equilibrium outcome. Note that in such a case, the outcomes under *independent scheduling* are all strictly worse for *Speaker*. On the other hand, we show that *status quo* may be sometimes less easily obtainable when T decreases, i.e. there may be a deadlock for some $T < n$, but no deadlock, for $T' < n$.

In the next proposition we provide a sufficient condition for *status quo* to be the unique equilibrium outcome. This is essentially a condition on how much competition there needs to be in order for it to force the outcome to 0.

Proposition 2. Let $y_0 = (-1, -1, ..., -1), y_i \geq 0, i = 1, ..., n$ and $T < n$. Suppose that for each $L \subset N, |L| = T$, there exists an $i \in L$ and a $j \in N \setminus L$ such that CC is satisfied for i and j. Then the unique pure-strategy equilibrium outcome is 0.

Proof. Suppose $\bar{m} \neq 0$ is an equilibrium outcome. There exist equilibrium proposals m^*, and a equilibrium set selected by agent 0, $L \subset N, |L| = T$, s.t. $\bar{m} = m_q(m^*, L)$. By assumption there exist an $i \in L$ and a $j \in N \setminus L$ such that CC holds.

Case 1: $y_{ij} < y_{ii}$ and $y_{ji} < y_{jj}$. Let m_i^* be i's equilibrium proposal. If $m_i^* > 0$, then there exists an $\epsilon > 0$, s.t. j would be better off proposing $m_j' = m_i^* - \epsilon$, which is impossible. Thus, $m_i^* = 0$, and $m_j^* \geq \max_{l \in L}\{m_l^*\}$. Since $\bar{m} \neq 0$, it follows that there is an $l \in L$, with $m_l^* > 0$, so that $m_j^* \geq m_l^* > 0$. But then, since $l \in L$, it follows that i would be better off proposing m_i', $0 < m_i' < m_l^*$, and would still be selected by the agent 0, which is a contradiction.

Case 2: $y_{ii} < y_{ij}$ and $y_{jj} < y_{ji}$. If $m_i^* > 0$, then if $m_j^* < \kappa$, i would be better off proposing $m_i' > m_j^*$ so that j's bid would be accepted. Moreover, $m_j^* = \kappa$ is impossible, since by assumption $\kappa > \max_{l \in N \setminus \{0\}} ||y_l||$, so that for every L', s.t. $j \in L'$, $m_q(L', m^*) = 0$. In other words, agent 0 would be better off choosing j's proposal since that would preserve status quo. Thus, $m_i^* = 0$, and we proceed as in Case 1. □

In the following two corollaries we provide easily-verifiable sufficient conditions on agents' preferences for a deadlock. The conditions are stronger if $T < n-1$, as compared to $T = n-1$. The reason behind this is combinatoric. If $T = n-1$, then it is enough that for each proposer i there be one other proposer with whom i competes for a slot on the agenda; if $T < n-1$ this is not enough, since then the pair can be excluded from the agenda and there is no reason for them to compete. Example 3 further illustrates this point.

Corollary 1. Let $y_0 = (-1, -1, ..., -1), y_i \geq 0, i = 1, ..., n$. Suppose $T = n-1$ or $T < \frac{n}{2}$, and suppose that for each proposer $i \in N$, there exists a proposer $j \in N \setminus \{i\}$, s.t. the pair i, j satisfies CC. Then the unique pure-strategy equilibrium outcome is 0, supported by $m_i^* = 0$, $\forall i \in N$.

Corollary 2. Let $y_0 = (-1, -1, ..., -1), y_i \geq 0, i = 1, ..., n$, and suppose $\frac{n}{2} \leq T < n-1$. Then 0 is the unique pure-strategy equilibrium outcome if either of the following conditions holds.

1. There exists a proposer $i \in N$, s.t. for each $j \in N \setminus \{i\}$, the pair i, j satisfies CC.
2. For each proposer $i \in N$, $|\{j \in N \setminus \{i\},$ s.t. the pair i, j satisfies CC$\}| \geq n - T$.

As we mentioned, if $\frac{n}{2} \leq T < n-1$ the conditions on preferences are a bit stronger for the uniqueness of 0-outcome. To round up this section, we provide an example, which illustrates this: it can indeed happen that smaller T, or *more competition*, makes Speaker worse off.

Example 3. Let $N = \{0, 1, 2, 3, 4\}$, $n = 4$, $y_0 = (-1, -1, -1, -1)$, and let the ideal points of proposers be given by $y_1 = (2, 1, 3, 3), y_2 = (1, 2, 3, 3), y_3 = (1, 1, 3, 2), y_4 = (1, 1, 2, 3)$. Then, if $T = 3$, by Corollary 1, the unique pure-strategy equilibrium outcome is 0.

On the other hand, observe that the ideal points of the agents do not satisfy the conditions in Corollary 2. Indeed, for $T = 2$, there exist non-zero equilibrium outcomes. For instance, it is easy to verify that $m_1 = m_2 = m_3 = m_4 = 3, L = \{3, 4\}$ is an equilibrium, with the outcome $m_q(m, L) = (0, 0, 3, 3)$.

6. ASSIGNMENT OF JURISDICTIONS

In this section we provide an example showing how the present model can be embedded in a richer setup. A natural question to consider in the context of the legislature is how the committees would want to choose their jurisdiction in an initial stage, when the set of jurisdictions is fixed. We use as the basic game the legislative game introduced in the preceding two sections.

Clearly, the assignment of jurisdictions matters, as each voter proposes a move only on one dimension, and it is conceivable that different assignments may facilitate different equilibria. In the next extended example we illustrate this fact. We take a simple scenario with 3 voters and *Speaker* and our aim is to analyze the equilibria of an extended game, in which the voters first choose their jurisdictions sequentially, and afterwards, the *legislative game* is played. Since this may happen according to various orderings of the voters, we first study all possible orderings, and describe the outcomes under every ordering. [12] Different agents may prefer different orderings, and it is not always true that an agent wants to move first - an agent may prefer to not be the most senior. As an aside, we also show that the condition in Proposition 2 is sufficient but not necessary.

Example 4. For the purpose of this example we enumerate the agents with Greek letters, because we will vary their jurisdictions and confusion may arise otherwise.

Let $n = 3$, $N = \{0, \alpha, \beta, \gamma\}$, let $T = 2$, and let the ideal points of the voters be given by $y_\alpha = (2, \frac{3}{2}, 1)$, $y_\beta = (1, \frac{3}{2}, 0)$, and $y_\gamma = (\frac{3}{2}, 0, 2)$. Each voter is assigned a jurisdiction, and each assignment of voters to jurisdictions is a one to one mapping from $\{\alpha, \beta, \gamma\}$ to $\{1, 2, 3\}$ and can be represented by an ordered triplet. For instance, the triplet (β, α, γ) represents the assignment where voter β proposes on dimension 1, α on dimension 2, and γ on dimension 3. We now show that different assignments foster different outcomes in the proposing and voting stages. Note that there are six possible assignments.

1. $\underline{(\alpha, \beta, \gamma).}$ Here the CC holds for α and β and for α and γ. To check for instance the first one observe that $y_{\alpha,1} > y_{\alpha,2}$ and $y_{\beta,1} < y_{\beta,2}$ which is CC since under this assignment α proposes on first dimension and β on the second one. Hence, Condition 1 of Corollary 2 holds, and the unique equilibrium outcome is 0.

2. $\underline{(\alpha, \gamma, \beta).}$ Now the CC only holds for γ, β and the sufficient condition in Proposition 2 is not satisfied. Nonetheless, the unique equilibrium outcome is 0. To see this, first note that in every equilibrium with non-zero outcome (and non-zero proposals) γ and β have to be selected - otherwise they would compete each other out by CC. Thus, $L = \{2, 3\} = \{\gamma, \beta\}$ and let $m^* = (m_1^*, m_2^*, m_3^*)$, $m_1^* > 0$, $m_1^* \geq m_2^*, m_3^*$. But no matter what the proposal m_3^* of agent 3 is, agent 2 always has an incentive to propose $m_2^* = 0$, and analogously for agent 3. This shows that the condition in Proposition 2 is sufficient but not necessary.

3. $\underline{(\beta, \alpha, \gamma).}$ CC holds for β, α and β, γ so that the unique outcome is 0 by Corollary 2.

4. $\underline{(\beta, \gamma, \alpha).}$ CC holds for β, γ and α, γ so that the unique outcome is 0.

5. $\underline{(\gamma, \alpha, \beta).}$ The CC holds only for γ, β, so that in every equilibrium with non-zero outcome it has to be that $L = \{1, 3\} = \{\gamma, \beta\}$. Since $y_{\beta,3} = 0$ we have that $m_\beta^* = 0$.

[12]In the US Congress, the assignment to committees is done according to seniority, hence a specific exogenously imposed ordering.

The equilibria are then given by proposals $m_\gamma^* = m_\alpha^* = k, k \leq \frac{3}{2}$, $m_\gamma^* = \frac{3}{2}, m_\alpha^* > \frac{3}{2}$, and in every subgame $L = \{\gamma, \beta\}$, so that the set of equilibrium outcomes is $\{(k, 0, 0) \mid k \leq \frac{3}{2}\}$.

6. $\underline{(\gamma, \beta, \alpha)}$. Again CC holds only for γ, β, so that $L = \{\gamma, \beta\} = \{1, 2\}$. The set of equilibrium outcomes is now given by $\{(k, k, 0) \mid k \leq \frac{3}{2}\}$. To see this observe that an outcome $(k, k, 0)$ is supported by bids $x_1 = x_2 = x_3 = k$, $\forall k \in [0, \frac{3}{2})$, and the outcome $(\frac{3}{2}, \frac{3}{2}, 0)$ is supported by bids $x_1 = x_2 = \frac{3}{2}$, $x_3 \geq \frac{3}{2}$. Notice that in the latter equilibrium agent γ obtains a payoff 0.

Now we analyze the extended game where the voters first sequentially choose their assignments, according to an exogenously imposed ordering (seniority), after which the proposal, selection, and the voting stages take place. For example, suppose the seniority is α, β, γ, α chooses the jurisdiction first, then β, and finally γ. If for instance the choices were $\alpha \to 1$, $\beta \to 3$, $\gamma \to 2$, then we are in Case 2 above and the final outcome is 0. Observe that such choices of jurisdictions are not subgame-perfect. We again look for the SPNE of the extended game, where before the three stages of the Legislative Game voters sequentially choose their jurisdictions according to the exogenously given seniority. One could analyze more complicated versions of the choosing-the-jurisdictions stage, but here we limit ourselves to a simple setup.[13]

There are 6 different seniority orderings in which the voters can choose their jurisdictions. Ordered lexicographically:

$$\sigma_1 = (\alpha, \beta, \gamma), \sigma_2 = (\alpha, \gamma, \beta), \sigma_3 = (\beta, \alpha, \gamma), \sigma_4 = (\beta, \gamma, \alpha), \sigma_5 = (\gamma, \alpha, \beta), \sigma_6 = (\gamma, \beta, \alpha)$$

Note that $\{\sigma_1, ..., \sigma_6\}$ is not the set of actual assignments of the jurisdictions but just the set of possible orderings in which the voters pick them, e.g. under σ_3, β is the first to pick, α the second, and γ last. Once the voters have picked their jurisdictions, we end up in one of the cases given by 1-6 above.

From the outcomes under 1-6 it is immediate that the incentives of α and β are aligned when they choose their assignments. Both prefer the outcome under 6 to that under 5 to everything else. However, γ prefers the outcome under 5 to everything else, since only under 5 he gets a higher utility than under *sq*. Thus, when γ chooses first he can get an outcome no better than 0, whereas if either β or α choose first γ may get a better outcome than 0.[14] Thus, having the highest priority in choosing the jurisdiction may not always be advantageous for γ, and he may be subject to a first-mover disadvantage. With this we conclude our example.

[13] One could even expand the game in which also Speaker is chosen by the legislature, or perhaps even the sizes of different jurisdictions, i.e., where some jurisdictions have more than one dimension. For this last exercise, one should first analyze more complicated versions of legislative game. Still, existence of equilibria is guaranteed in such games by Theorem 1.

[14] Indeed, if we apply for instance a forward induction refinement, β and α should always choose an outcome which leads to Case 5 above, whenever either of them moves before γ.

7. DISCUSSION

In the present paper we have provided the general framework of the *scheduling game* in the legislative process. A very natural application is to provide a more realistic model of the legislative process where there is scarcity of time on the one hand, while on the other hand, there is an agent wielding large agenda powers. We have formulated the *scheduling game* with this application in mind, especially with the voting outcome function mapping the selected proposed moves from the *status quo* into final outcomes.

While there is a considerable amount of complication that arises in the *scheduling game*, it provides a very rich framework for the analysis of political process. Moreover, in the *scheduling game* many of the standard problems of voting models do not arise. First and foremost, the existence of equilibria is assured by our general existence theorem. Second, for the more specific case of the *legislative game*, the equilibria are relatively well behaved, and one does not run into the problems such as "Chaos theorems" (McKelvey 1976; see also Plott 1967) . Thus, a rational-choice model, built on purely non-cooperative foundations, can provide well calibrated insights into the legislative process. Finally, the general model *scheduling game* provides other possible formulations of the committee-bargaining problem, in particular, the jurisdictions of committees in practice overlap. In traditional models, such overlaps cause problems with existence of equilibria, while in the present model, no such problem arises. Modified *scheduling game*, where the outcome function is not a voting function, but simply the choice function of the auctioneer (*Speaker* in the present model) might also provide a useful model for other applications in Political Economy.[15]

An issue that arises in our model is the power delegated to Speaker. It is important to observe that this power is weaker than a veto power, since in our model, Speaker *has to* choose T proposals out of n, and cannot choose less. When Speaker can commit *ex ante*, before any of the proposals are submitted, to choosing a precise number of slots on the agenda, this describes precisely the situation arising in our model. However, if Speaker is allowed to decide how many proposals to choose *once he observes the set of proposals* this would qualitatively change the set of equilibria. Clearly, Speaker would wield a much greater power since he would then always be able exclude any moves contrary to his preferences. At the same time, the incentives to proposers would also change. In particular, deviations from equilibrium strategies might generate different sets of outcomes, e.g., if there are 2 proposers both proposing something unfavorable to Speaker, he could simply choose nothing. In short, the set of equilibria of a model where

[15]For instance, think of a government auctioning off a contract for developing a public project. Clearly, the government cares not only about the revenue that it raises, but also about some other parameters of *quality* of the development. On the other hand, the competitors for the project do not care only about whether they get the project or not, but also who gets the project and how many resources they would have expend, as this will affect the ability of this contractor to compete for other projects. One such example are the spectrum auctions: there, the government cares about raising revenue, as well as about the concentration of the industry, the subsequent competition in the product market, and investment in R & D.

Speaker is allowed to pick any number of proposals after they are revealed to him is most likely going to be different from the union of the set of equilibria in our model - the union taken over the number of proposals that Speaker has to pick.

An interesting extension of the present model would be to account for incomplete information. For instance, perhaps the simplest specification of the *legislative game* with incomplete information is one where only y_0 is private information to *Speaker*, and each voter obtains a signal about it, e.g., the component corresponding to his jurisdiction. Another relatively simple incomplete-information version of *legislative game* is one where incomplete information is on *status quo* - that is *status quo* embodies a resolution of some randomness. One would need to simplify agents' preferences, but our belief is that such models might ultimately prove very useful.

We stress that this model of scheduling developed is meant only as more plausible representation of the legislative process that is used in practice in most democratic legislatures. It is an open question as to why so many legislatures have adopted such a system to allocate scarce plenary time. Our model, for example, is subject to Riker's (1980) critique of legislative design: why would a majority of legislators delegate this scheduling authority to the *Speaker* since this will allow her to expropriate significant policy rents? Clearly, this would require a complete model of optimal legislative design, which is beyond the scope of this paper. However, our general model may provide the initial framework for such analysis.

A. PROOFS

A.1. *Theorem 1*

Proof. The proof follows from the main Theorem of Simon and Zame(1990). To show that, we follow their discussion on the last paragraph of p. 864 of their paper.

Construct a set S^* as follows. Take a $q \geq \frac{1}{2}, T \leq N$, and let for each $L \subset N$, s.t. $|L| = T$,

$$C_T^+(L) = \{m \in M \mid m_q(m, L) \succ_0 m_q(m, L'), \forall L' \neq L, |L'| = T\},$$
$$C_T^-(L) = \{m \in M \mid 0 = m_q(m, L) \succ_0 m_q(m, L'), \forall L' \neq L, \text{ s.t. } |L'| = T, \text{ and } m_q(m, L') \neq 0\}.$$

Define

$$C_T(L) = C_T^+(L) \cup C_T^-(L),$$

and

$$S^* = \cup_{L \subset N, |L| = T} C_T(L).$$

Intuitively, $C_T(L)$ is the set where *Speaker* strictly prefers picking the subset of proposals L over any other subset of proposals, and S^* is the set of proposals where he has a strict preference for choosing some set of proposals. By continuity of *Speaker*'s preferences, there exists for every point $m \in S^*$ a neighborhood Γ_m, Γ_m open in S, $\Gamma_m \subset S^*$, and such that *Speaker* selects the same subset of proposals on the whole Γ_m (i.e. if $m \in C_T(L)$ then $\Gamma_m \subset C_T(L)$). Thus, the subgame-perfect outcome correspondence $\phi : S^* \to M$ is continuous on S^*.

The set S^* is dense in $S = M_1 \times M_2 \times ... \times M_n$. The reason is local non-satiation of *Speaker*'s preferences almost everywhere (except at his ideal point). Thus, ϕ can be extended to an upper hemi continuous, bounded, and convex-valued correspondence Q on S. To see that such Q is subgame perfect note that whenever there is no indifference for *Speaker* in the subgames, it has to be that $m \in S^*$, and whenever $m \in S \setminus S^*$, *Speaker* can resolve his indifference either way, or by picking the top sets of proposals probabilistically. Note that in equilibrium the Speaker resolves his indifference in precisely the right way. Therefore, by applying backward induction the game in stage 1, with the outcome correspondence Q satisfies assumptions in Simon and Zame(1990), and an equilibrium exists. Hence a subgame-perfect equilibrium of the original game exists, in which *Speaker*'s resolution of his indifference is specified as a part of the equilibrium.

□

A.2. *Proposition 1*

Proof. Let $T < N$. Let (m^{*I}, L^I) be the vector of equilibrium proposals and the set of selected voters under IS. Now define a new legislative game Γ', where the set of feasible proposals is given by

$$M_i = \{m_i \mid m_{i,i} \in [\underline{b}_i, \bar{b}_i],\ m_{i,j} = 0, j \neq i\},$$

for each $i \in N$. We will now appropriately specify the constants $\underline{b}_i$ and $\bar{b}_i$.

For each $i \in N \setminus L^I$ let $\underline{b}_i = -K, \bar{b}_i = K$, where recall that K is the constant bounding the proposals in the definition of the original scheduling game. For each $i \in L^I$ proceed as follows. If $m_i^{*I} \neq y_{0,i}$ then define $m_i' \neq m_i^{*I}$ such that $m^{*I} \sim_0 m'^I$, where $m_j'^I = m_j^{*I}$, $\forall jnei$ and $m_i'^I = m_i'$. If $m_i^{*I} = y_{0,i}$ then let $m_i' = m_i^{*I}$. Finally, let $\{\underline{b}_i, \bar{b}_i\} = \{\min\{m_i^{*I}, m_i'\}, \max\{m_i^{*I}, m_i'\}\}$.

By Theorem 1 there exists an equilibrium of the legislative game Γ'. Denote the equilibrium proposals by $m^{A'}$, and by $L^{A'}$ the selection rule of agent 0. We show that *either*:

(i) $m^{A'}$ and $L^{A'}$ also constitute an equilibrium of Γ, *or*

(ii) There exists another equilibrium (m^*, L^*) of Γ in which Speaker is better off.

Clearly, it is enough to prove (ii). We need the following.

Lemma 1. Let $(m^*, L^*(.))$ be an equilibrium of Γ, $L^*(m^*) = \bar{L}$, s.t. $a_q(m^*, \bar{L}) \neq 0$. Let $(m^{*I}(\bar{L}), \bar{L})$ be the IS equilibrium when $\bar{L}$ are the selected voters. If $a_q(m^*, \bar{L})) \neq a_q(m^{*I}(\bar{L}), \bar{L})$ then there exists a voter $j \in N \setminus \bar{L}$ and a voter $i \in \bar{L}$, s.t. $a_q(m^*, \bar{L}) \sim_0 a_q(m^*, \bar{L} \setminus \{i\} \cup \{j\})$.

Proof. The proof follows from the additive separability of voters' preferences: if the outcome is not 0, then regardless of what the outcome is in other dimensions, in his own dimension a voter always prefers to bid his ideal point. Formally, assume $(m^*, L^*(.))$ is a SPNE of the legislative game, and assume $a_q(m^*, \bar{L})) \neq a_q(m^{*I}(\bar{L}), \bar{L})$, i.e., the outcome in this equilibrium of the legislative game is different from the outcome under independent scheduling when the same voters are selected. Therefore, at least on of the voters, say $i \in \bar{L}$ must be in the legislative game making a proposal which he is not making under the IS. By additive separability, it is clear that this voter would have preferred to make the same proposal as under IS, and by local strict monotonicity of i's preferences, it must be that $\exists j \in N \setminus \bar{L}$ which would make speaker indifferent to selecting i's proposal or j's proposal. This concludes the proof of the lemma. □

Now assume that $(m^{A'}, L^{A'})$ is not an equilibrium of Γ. Thus, there exists an agent $i \in L^{A'}$ who can deviate in Γ to make a different proposal which would get selected, and which would change the oucome. But the only such possible deviation is in order to make 0 be the outcome of the voting stage, by above Lemma 1. Since i would get selected in Γ, this implies that 0 is preferred by Speaker to $a_q(m^{A'}, L^{A'}(m^{A'}))$ in Γ. $\square$

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